A Practical Application of Relativity to Observation and Control of MESSENGER Timekeeping

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The MESSENGER mission to Mercury, designed, implemented, and managed by The Applied Physics Laboratory, uses a partially automated timekeeping system to ensure that the knowledge of time is properly maintained onboard the spacecraft and on the ground. Since 2007, accurate time has been maintained when required through the use of a very stable oscillator. The behavior of that oscillator, and hence of the timekeeping system itself, has varied in flight in a manner that was not observed on the ground but which conforms to the special and general theories of relativity. Through accounting for the effects of relativity, we have been able to determine the rate of aging of the oscillator and are therefore able to predict the future behavior of the timekeeping system. This knowledge is now routinely applied by the MESSENGER Mission Operations Center to predict when new time parameters will need to be uploaded to the spacecraft as well as the values that need to be uploaded.

INTRODUCTION

The MErcury Surface, Space ENvironment, GEochemistry and Ranging (MESSENGER) mission timekeeping system provides accurate time information to both onboard and ground-based users of MESSENGER time.[1] In particular, it provides real-time estimates of time to the onboard guidance and control (G&C) subsystem, which needs knowledge of time to support the critical function of spacecraft attitude control. The accuracy of the timekeeping system, however, depends on the performance of the oscillator that is used to drive the onboard clock counter. That counter is called the mission elapsed time (MET) counter and records elapsed time in terms of clock "ticks," each such tick lasting approximately 1 μs.

Time is provided to the G&C subsystem in two forms, namely, MET and Terrestrial Dynamical Time (abbreviated TDT or TT). Terrestrial Dynamical Time is a representation of time on the surface of the Earth and, for practical purposes, is equivalent to the commonly used UTC (Coordinated Universal Time). This estimate of Earth time is used by the G&C subsystem to determine where the spacecraft is in its orbit so that it can determine what the spacecraft attitude should be.

The spacecraft actually contains four oscillators that can be used to support onboard timekeeping. The current configuration of the spacecraft avionics allows either of two of those oscillators to be used to drive the MET counter. One is a very stable oven-controlled crystal oscillator (OCXO) that is used to provide the timekeeping system accuracy required to support science observations. Owing to reliability concerns, however, the other less-stable "coarse" oscillator is used to drive the MET counter whenever an engine burn is performed. This has occurred numerous times—during cruise, during Mercury orbit insertion (MOI), and also at many solar perihelia and aphelia while the spacecraft is in orbit around Mercury. This occasional deselection of the "precision" oscillator, the OCXO, has produced numerous gaps in our record of OCXO performance. Even with those gaps, however, some very interesting and useful information has been gathered about the behavior of the OCXO and of the timekeeping system itself.

The timekeeping system produces, on the ground, an estimate of the correlation between MET and Earth time in terms of the TT system. It saves that estimate in a text file called the Operations SCLK (Spacecraft Clock) Kernel. The SCLK Kernel estimate of clock behavior directly maps to average oscillator behavior. When the OCXO is selected, comparison of that average oscillator behavior to how we would predict the oscillator to behave according to the special and general theories of relativity provides us with knowledge of how much the OCXO behavior deviates from those predictions. Those "residuals" tell us how the physical OCXO actually behaves, as a local observer on the spacecraft would see it. Trending those residuals allows us to predict how the OCXO will behave in the future, and that prediction, in turn, is routinely used by the MESSENGER Mission Operations Center at APL to control the onboard real-time estimate of Earth time (TT) used by the G&C subsystem.[2]

The MESSENGER timekeeping system has not only provided us with the practical result of accurate onboard time for the G&C subsystem but has also provided a rare opportunity to have a glimpse, however small, into the theoretical universe of Albert Einstein.

THE VENUS FLYBY IN 2007

In early 2007, the MESSENGER spacecraft began to use the oscillator designated OCXO-B to control the spacecraft clock. Shortly afterward, the spacecraft had its second flyby of Venus to provide a gravity-assisted course correction on its long journey toward Mercury. The variation in OCXO behavior that was evident from the behavior of the MET counter as estimated in the Operations SCLK Kernel was much greater than we had seen during testing on the Earth and was much greater than could be explained by environmental factors such as the sensitivity of the OCXO output frequency to temperature variations. The explanation lies in the theory of relativity, which tells us that both velocity and gravity can measurably affect the observed behavior of a distant clock or oscillator. Time, according to the theory, is not the same for all observers.

Observations during the 2 months after the OCXO was selected, including the closest approach to Venus during the second MESSENGER flyby of Venus, are illustrated in Fig. 1. The observed OCXO fractional frequency offset (bottom curve) varied by 11 parts per billion (ppb), much more than we had observed during ground testing. That is equivalent to a variation in the MET counter of 11 ns/s or 0.95 ms/day. This OCXO was custom designed to limit sensitivity to environmental factors and, for example, was designed to allow only 4 ppb OCXO fractional frequency variation or 0.35 ms/day over the full 40°C operational temperature range. Also, the oscillator experienced a temperature variation of only 4°C during the Venus flyby. The frequency changes that we observed were much greater than environmental variations could cause.

Let's focus first on the effect of velocity on the oscillator frequency performance. Special relativity tells us that time on the spacecraft (the MET counter) is related to the time perceived by a distant observer by the Lorentz factor $\gamma = 1/[1 - (v^2/c^2)]^{0.5}$, where c is the speed of light and v is the magnitude of the velocity of the spacecraft in the coordinate frame of the distant observer. Because oscil-

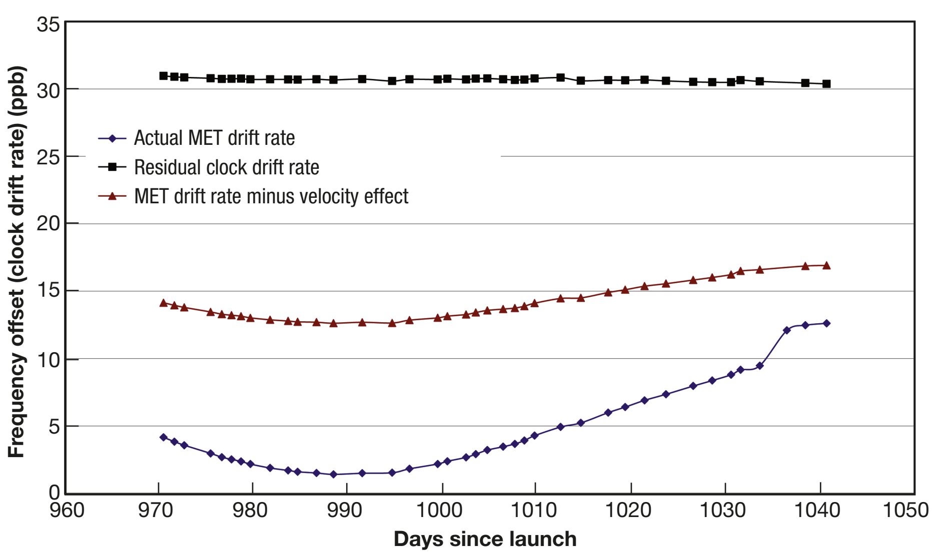

Figure 1. Residual MESSENGER OCXO-B behavior after March 2007 turn-on.

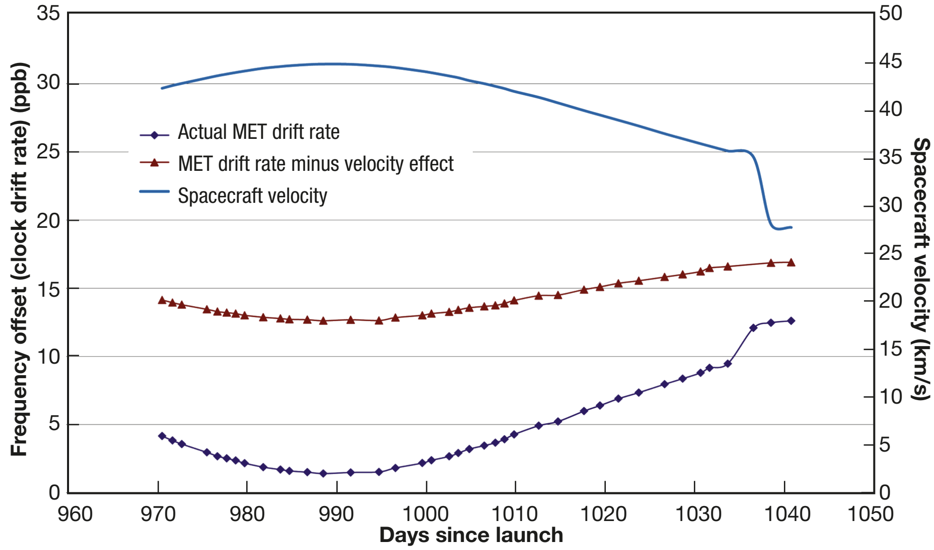

Figure 2. MESSENGER OCXO-B behavior after March 2007 turn-on, with relativistic velocity effect removed.

lator frequency and clock time are closely related, the Lorentz factor also describes how our observation of OCXO frequency is related to the oscillator frequency that would be seen by an observer traveling with the spacecraft. Subtracting this effect from our data, the bottom curve in Fig. 1, leads to the middle curve. All computations are with respect to the solar barycenter, that is, the velocity is the velocity of the spacecraft according to an observer at the solar barycenter.

A particularly interesting feature of these curves is that the jump in OCXO fractional frequency offset that appears near the end of the observed data is absent from those data after removal of the relativistic velocity effect. That jump corresponds to the simultaneous reduction in velocity, shown in Fig. 2, which resulted from the close flyby of Venus. (The apparent time shift in the observed jump in OCXO frequency compared with the time of the jump in velocity is an artifact of the way the frequency jump was computed, but that detail is discussed later.)

The general theory of relativity provides the tools for understanding how gravity affects the output frequency of the OCXO. The stronger the gravity field experienced by the oscillator, the slower that oscillator runs, at least as seen by a distant observer in a fixed gravity field. That behavior is described by the expression $-(GM/c^2)/R$, where G is the gravitational constant, M is the mass of the body that is the source of the gravity, c is again the speed of light, and R is the distance from that same body. For Fig. 1, M is the mass of the Sun, and R is the distance of the spacecraft from the Sun. The gravitational effect of Venus was also examined but found to be negligible on the scale shown in the figure. The residual OCXO drift rate shown, the top curve in Fig. 1, is the result of subtracting both the gravitational effect and the velocity effect from the observed data. The variation of residual frequency offset from 31.0 ppb to 30.4 ppb over 70 days is reasonable and can be explained by a combination of temperature effects and oscillator frequency aging.

This tells us that, after removal of the relativistic effects, the OCXO was running fast by approximately 30–31 ppb, approximately 2.6–2.7 ms/day.

TDTRATE AND THE MERCURY FLYBYS

After the 2007 flyby of Venus, the MESSENGER spacecraft flew past Mercury three times and then, in a fourth encounter, went into orbit around Mercury. From the Venus flyby, we were able to infer from the

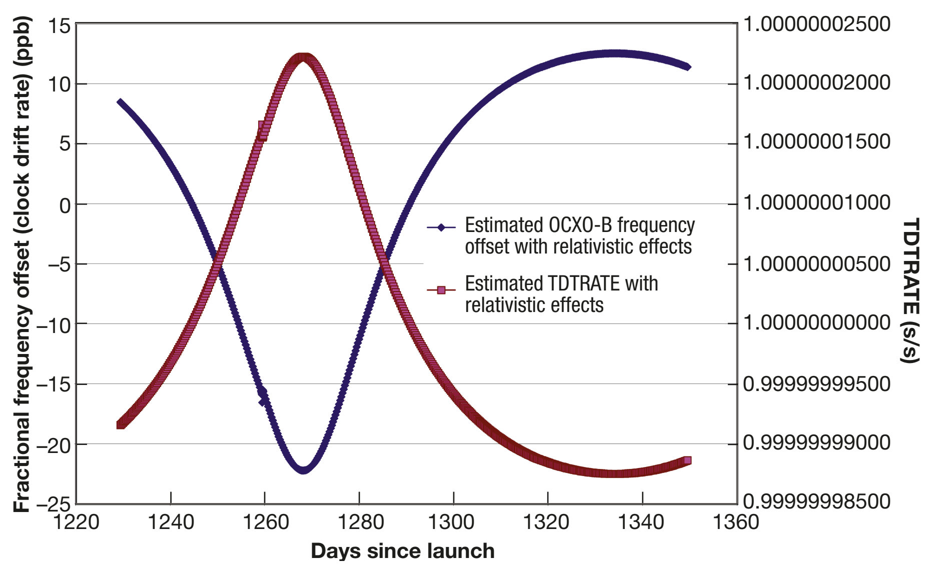

Figure 3. Estimated MESSENGER OCXO-B fractional frequency offset and TDTRATE during M1 due to relativity, assuming +31 ppb offset without relativity.

behavior of the MET counter how the OCXO behaved. We also determined that, with relativistic effects removed, the OCXO was running fast, that is, the OCXO output frequency was higher than the nominal design frequency.

Prior to the first encounter with Mercury (M1), we reversed that process and, with the assumption that the OCXO was still running fast by the amount determined from the Venus data, derived the expected behavior of the OCXO and of the MET counter. That result is shown in Fig. 3, calculated using spacecraft velocity relative to the Sun and the gravitational effects of both the Sun and Mercury. One curve is the OCXO output frequency and equivalent clock drift rate expressed in parts per billion. Note how the oscillator and clock slowed down as solar perihelion was approached. The other curve expresses the clock behavior in terms of the parameter TDTRATE, an important metric of interest to Mission Operations Center staff. Note that the observations of clock behavior during the 2007 Venus flyby were actually observations of TDTRATE that were converted to the equivalent oscillator frequency offset behavior shown in Figs. 1 and 2.

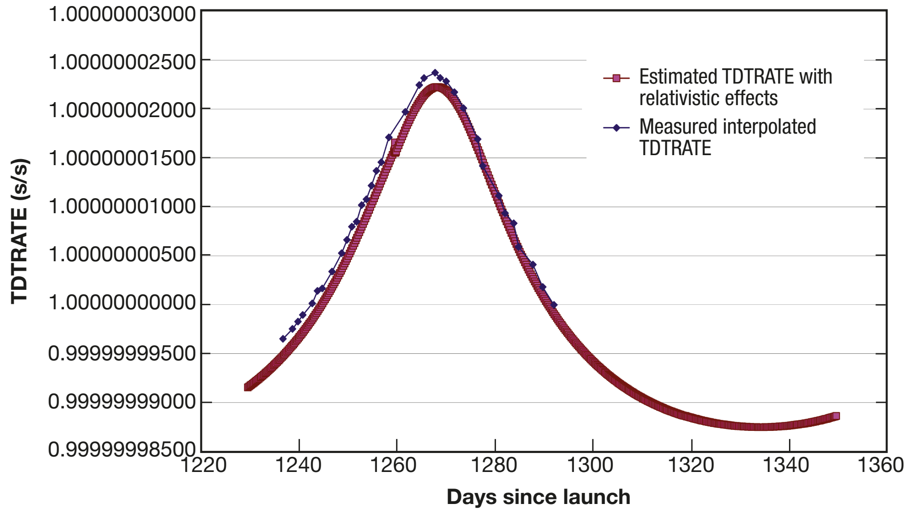

Figure 4. Estimated MESSENGER OCXO-B TDTRATE during M1 due to relativity, assuming +31 ppb OCXO fractional frequency offset without relativity, compared with actual TDTRATE.

TDTRATE is obtained from the Operations SCLK Kernel. The Jet Propulsion Laboratory's Navigation and Ancillary Information Facility (NAIF) provides to APL and to other users a suite of software and information system tools called Spacecraft Planet Instruments C-matrix Events (SPICE), designed to provide access to information about space missions. One component of SPICE is a set of "kernels" or parameter files that contain information relative to specific missions or to space missions in general. For example, ephemerides kernels might provide the ephemerides of planets or of specific spacecraft. One type of SPICE kernel is the SCLK kernel, a text file that provides mappings between MET and Earth time. Each time record of the SPICE SCLK kernel contains three fields: (1) a field equivalent to MET, (2) a field representing Earth time, and (3) a clock change rate. At APL, our SCLK kernels have always expressed Earth time in the TT time system (previously abbreviated TDT), and we have typically called the clock change rate TDTRATE.

The standard JPL SPICE SCLK kernel defines the clock change rate as a rate to be used to extrapolate the current MET–TT mapping to future times. At APL, we describe that rate as a "predicted TDTRATE." Predicted TDTRATE is used, for example, in predicting the future time of execution of a command and is used to provide real-time estimates of time to the G&C subsystem. However, the APL-defined Operations SCLK Kernel takes that a step further and recalculates TDTRATE

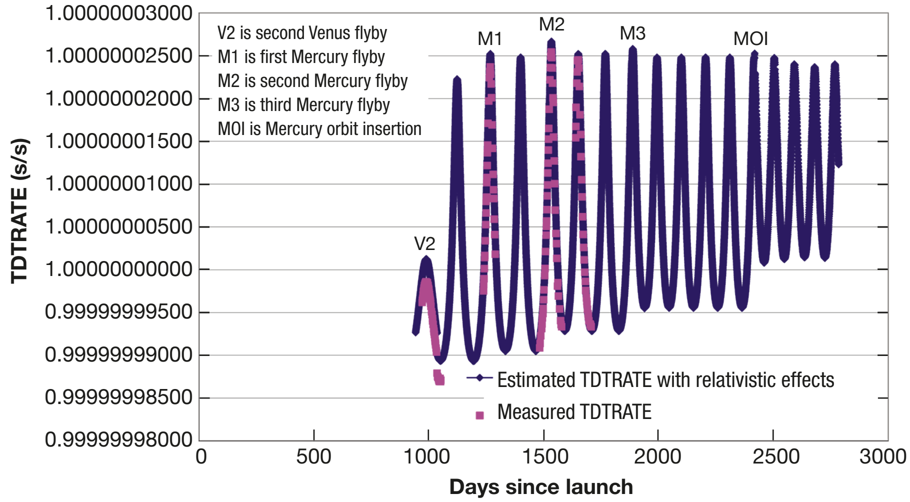

Figure 5. Estimated MESSENGER OCXO-B TDTRATE, including the effects of relativity, assuming +28 ppb OCXO fractional frequency offset without relativity, using orbit determination solution OD153.

for past times to provide a more accurate mapping between MET and TT. That improved mapping is needed to provide the time accuracy required for analysis of science observations. That interpolated TDTRATE is shown in Fig. 3. The interpolated TDTRATE is based on both the current time record in the SCLK kernel and on the next time record, so the computed TDTRATE and derived oscillator frequency "lead" the actual performance of the oscillator. That is why the change in oscillator frequency shown in Fig. 2 appears to occur slightly before the actual change in spacecraft velocity during the Venus flyby.

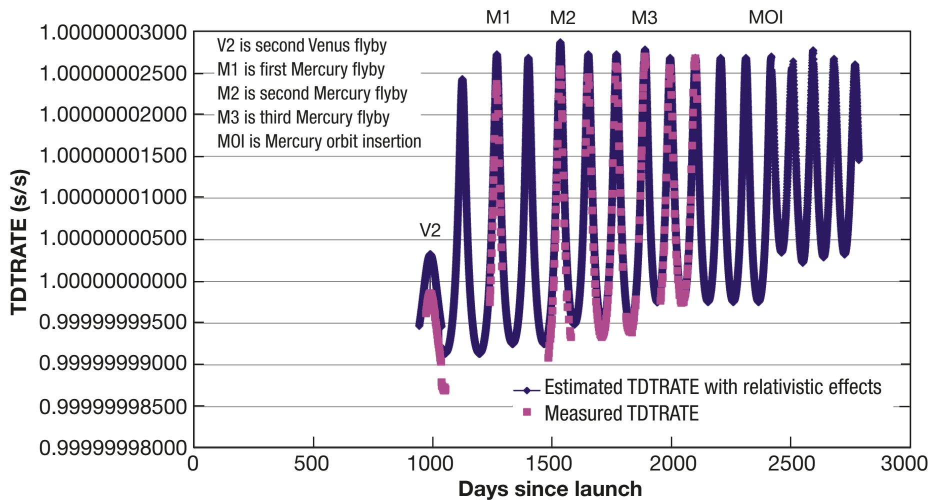

Figure 6. Estimated MESSENGER OCXO-B TDTRATE, including the effects of relativity, assuming +26 ppb OCXO fractional frequency offset without relativity, using orbit determination solution OD179.

A comparison between the interpolated TDTRATE taken from the Operations SCLK Kernel after the M1 flyby and the relativistic prediction of TDTRATE given in Fig. 3 is shown in Fig. 4. The comparison is fairly good but not perfect and suggested that the +31 ppb OCXO fractional frequency offset of the Venus flyby might need to be adjusted.

Analysis of TDTRATE following the second flyby of Mercury (M2) provided a better fit of +28 ppb to the M1 and M2 data, after relativistic effects were removed. However, that same analysis did not provide a good fit to the Venus flyby data, as can be seen in Fig. 5. The results shown in this figure provided the first prediction of OCXO TDTRATE through the end of the primary orbital phase of the mission, following a full Earth year in orbit around Mercury, and was calculated using spacecraft velocity relative to the Sun and the gravitational effects of the Sun, Earth, Venus, and Mercury. The gravitational effect of distant Jupiter is insignificant at this scale. This figure was created using an orbit determination designated OD153 by our Navigation Team at KinetX, as noted in the figure; later orbit determinations were used for the later figures. Note that TDTRATE has local maxima at solar perihelia, indicating that the OCXO output frequency and the clock drift rate have local minima near perihelia. Similarly, TDTRATE has local minima at aphelia, and the OCXO frequency and clock drift rate have local maxima at aphelia. In other words, the clock is running slower at perihelia (larger TDTRATE) and faster at aphelia (smaller TDTRATE), due to the relativistic effects of velocity and gravity.

THE USE OF RELATIVITY TO CONTROL ONBOARD TIME

It was noted earlier that the onboard G&C subsystem uses an estimate of Earth time. At various times, the Mission Operations Center uploads new time parameters to the spacecraft to allow the G&C subsystem to compute an accurate estimate of TT from MET. One of those parameters is a recently calculated value of predicted TDTRATE. Figure 5 and the calculations underlying the predictions of TDTRATE in that figure were

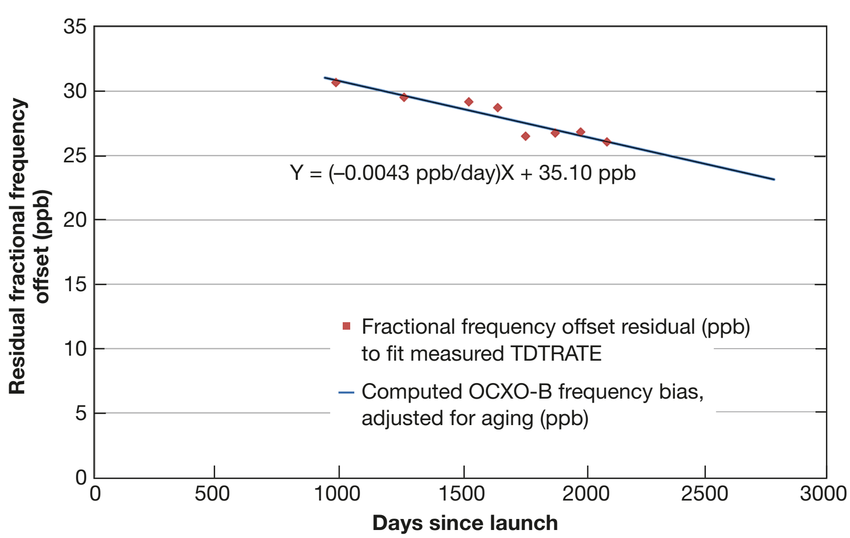

Figure 7. First linear fit to residual OCXO-B fractional frequency offset.

provided to James Hudson of the MESSENGER Mission Operations Center in April 2009. As described by Hudson and Colwell,[2] the Mission Operations Team used that information to create a model to determine an optimal value of TDTRATE to use onboard for the third Mercury flyby (M3) a few months later. The success of that process is shown in Fig. 5 of Ref. 2. Although relativistic corrections had been used for spacecraft before, such as for the GPS constellation,[3] this was a significant event for APL, perhaps the first time that a relativistic prediction had been used to control the accuracy of time onboard an APL spacecraft.

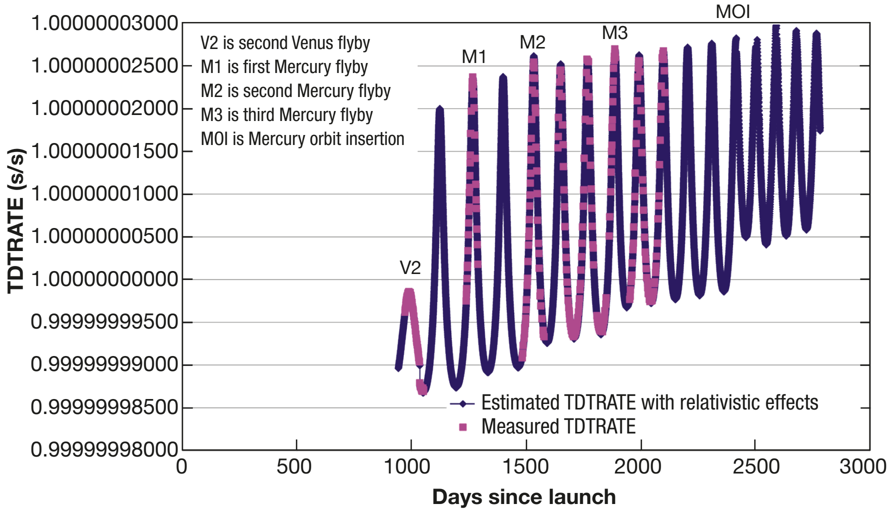

Figure 8. Estimated MESSENGER OCXO-B TDTRATE, including the effects of relativity, assuming −0.0043 ppb/day OCXO linear frequency aging, using orbit determination solution OD179.

NEW INSIGHTS INTO OSCILLATOR BEHAVIOR

The apparent change in fractional frequency offset from the second Venus flyby (Fig. 1) to the second Mercury flyby (Fig. 5) suggested that the precision oscillator (OCXO) might be slowing down. That trend continued through the third Mercury flyby in September 2009; see Fig. 6. At that point, the characteristics of aging of that particular oscillator were not known other than the prelaunch vendor estimate of less than ±0.05 ppb/day for normal oscillator aging exclusive of any radiation effects. Using available values of interpolated TDTRATE taken from the Operations SCLK Kernel for times near eight solar perihelia, as shown in Fig. 6, the relativistic effects were removed and a linear least squares fit was applied to those residuals, shown in Fig. 7. This procedure gave an estimated average aging rate of −0.0043 ppb/day, much better than expected from the vendor's estimate.

The linear least squares fit of Fig. 7 was applied to the data of Fig. 6 to yield a prediction of TDTRATE with linear aging, which is shown in Fig. 8. With that estimate of OCXO aging, the relativistic prediction of TDTRATE "snapped into focus." Over the following year, the performance of this estimate continued to be monitored (and the results provided to the Mission Operations Team), but the same linear least squares fit continued to be applicable. The OCXO fractional frequency offsets corresponding to Fig. 8 are shown in Fig. 9. All the flybys occurred at perihelia, where the oscillator frequency was lower and the clock was running slower.

Near the end of the primary orbital phase of the mission in March 2012, a new linear least squares fit was made using only TDTRATE observations from the one year in orbit from March 2011 to March 2012, and the results were provided to the Mission Operations Team to support maintenance of onboard G&C time for

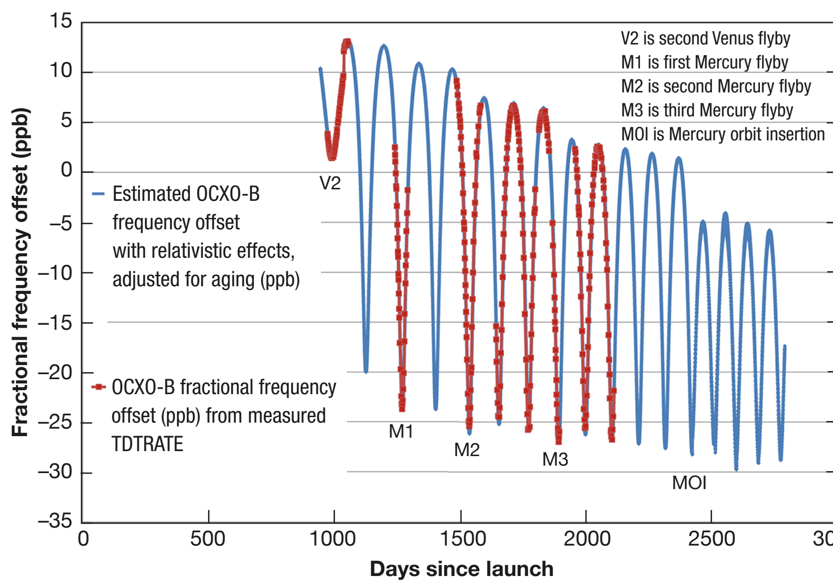

Figure 9. Estimated MESSENGER OCXO-B fractional frequency offset, including the effects of relativity, assuming −0.0043 ppb/day linear frequency aging, using orbit determination solution OD179.

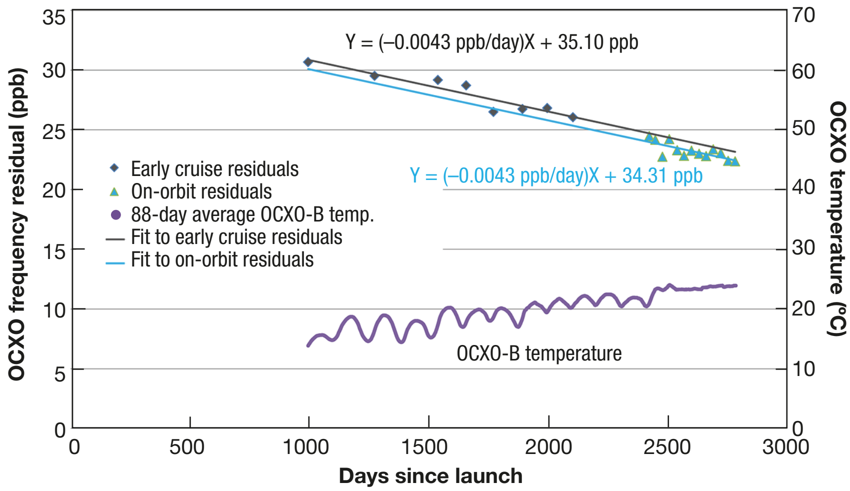

Figure 10. OCXO-B fractional frequency residuals for last 5 years of the MESSENGER primary mission, after correction for relativistic effects, using orbit determination solution OD260.

the extended mission that began on 18 March 2012. The results were somewhat surprising. The residuals (Fig. 10) again exhibited OCXO aging at a linear rate of −0.0043 ppb/day, but the linear fit was offset by almost −0.8 ppb from the earlier linear fit of cruise residuals (Fig. 7). Although it remains unclear, even after examination of additional data, whether the change in OCXO frequency was sudden or gradual, it appears likely that the increased OCXO temperature in orbit, shown in Fig. 10, accounts for at least part of the 0.8-ppb shift.

CONCLUSIONS

We've observed that the behavior of the MESSENGER precision oscillator, an OCXO, and the behavior of the clock driven by that oscillator are strongly influenced by relativistic effects due to the dynamics of the spacecraft trajectory. We successfully employed that observation to predict the future behavior of the onboard clock. The MESSENGER Mission Operations Team has incorporated that information into routine operations in determining the parameters needed to compute onboard G&C time. This has resulted in an improvement in operator control of the error in G&C time.

It was also determined, using knowledge of these relativistic effects, that the OCXO ages approximately linearly when the average oscillator temperature is fairly steady and that a linear fit to TDTRATE residuals provides a good prediction of future oscillator and onboard clock behavior.

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