Pergamon

0005-1098(95)00002-X

Automatica, Vol. 31, No. 6, pp. 913-915, 1995
Elsevier Science Ltd
Printed in Great Britain
0005-1098/95 $9.50 + 0.00

Technical Communique

On the Use of an SPSA-based Model-free Controller in Quality Improvement*

FAHIMEH REZAYAT†

Key Words—Quality control charts; adaptive control; stochastic approximation.

Abstract—There exists a growing realization that quality will be gained by implementing statistical process control and engineering process control in a complementary fashion. This study continues in that direction by considering the batch polymerization example of Vander Wiel *et al.*, but it assumes no knowledge about the dynamics of process. It uses concurrently the special-cause control chart and the simultaneous perturbation stochastic approximation (SPSA)-based control approach proposed by Spall and Cristion to monitor, signal and readjust the levels of viscosity of simulated batches of polymer around the target value. This study also compares the performance of the SPSA-based adaptive control and one-step feedback controller when the process dynamics changes.

1. *Introduction*

In recent years the importance of quality has become increasingly apparent. Quality control in manufacturing has moved from detecting nonconforming products through inspection to continuously reducing variability in product performance and production process. Two existing fields that have been contributing to quality are statistical process control and engineering process control. There has been a growing realization that both of these approaches should be implemented in a complementary fashion, and that quality will be gained through appropriate process adjustment and through elimination of the root cause of variability (see e.g. Vander Wiel *et al.*, 1992; Box and Kramer, 1992; Box, 1993). This paper continues in that direction, with one critical difference: it assumes that *no knowledge* about the exact process dynamics exists, and uses, concurrently, the special-cause control (SCC) chart (Alwan and Roberts, 1988)—a statistical process control technique, for monitoring and signalling out of control process—and the SPSA-based neural network (NN) controller proposed by Spall and Cristion (1992, 1994)—an adaptive control technique.

The NN controller uses the simultaneous perturbation stochastic approximation (SPSA) technique for training the NN; it is appropriate when the system can tolerate non-optimal control (training process) and the regularity conditions in Spall (1992) are met; it is novel in that it employs only one NN as a controller, and does not need information about the process dynamics or a second NN to model the process (as needed in a back-propagation-type approach). The unique attribute of SPSA is that it can efficiently solve an optimization problem without requiring detailed information about the characteristics of the system. SPSA is based only on noisy measurements of the objective function, and does not require direct gradient or higher-derivative computation. The main input at each iteration of SPSA is an approximation to the gradient vector that is based on two measurements of the objective function, independent of the problem dimension p (i.e. the number of the parameters to be optimized).

The belief is that most often the exact relationship between the value of inputs and outputs is unknown to quality control practioners; or, even if it is known in principle, owing to its complicated nature or to complicated boundary conditions, it may be virtually impossible (or at least very expensive) to compute. In practice, after the control factors (critical inputs or inputs whose level can be manipulated) are identified, the adjustment of the level of control factors are often done subjectively by experts. It is believed that using the SPSA-based NN controller to adjust the level of control factors, along with statistical process control charts for monitoring output and signalling out-of-control process, leads to better output quality, especially when the process design is relatively new and/or an expert is not available.

This work is continuation of the earlier work by Rezayat (1993), with the emphasis here being on comparing the performance of the NN controller with the one-step feedback controller when the process dynamics changes, and on using statistical process control, concurrently with adaptive controllers, to monitor and signal the output variations around the target value.

The remainder of the paper is organized as follows. Section 2 reviews briefly the SPSA/NN-based adaptive controller employed in this study, and statistical control charts. Section 3 provides simulation experiments and analysis of their findings related to reducing a polymer's viscosity deviations around a target value (using the example given by Vander Wiel *et al.*, 1992).

2. *Preliminaries*

2.1. *A brief review of the SPSA-based NN controller.* To design a NN, in general, one has to determine the number of nodes and layers, and iteratively estimate its parameters (weights). The goal of the weight estimation process is to find a vector of weights that results in a sequence of control values that drive the process output close to the target value, given knowledge of the current state of the system. Existing literature estimates the NN weights Θ_k using a stochastic approximation algorithm of the form

$$\hat{\Theta}_k = \hat{\Theta}_{k-1} - a_k \hat{g}_k(\hat{\Theta}_{k-1}), \quad (1)$$

where $\hat{\Theta}_k$ denotes the estimate of Θ_k at the kth iteration, a_k is a scalar gain sequence satisfying certain conditions, and $\hat{g}_k(\cdot)$ is an estimate or observation of the gradient of the loss function. Traditional NN controllers use standard gradient-based search techniques to estimate NN weights (see e.g. Narendra and Parthasarathy, 1990), and the value of $\hat{g}_k(\cdot)$ requires knowledge of the process dynamics (say, as in back-propagation).

When the process dynamics are unknown, one cannot implement standard gradient-based search techniques. Spall

* Received 19 April 1994; revised 15 August 1994; revised 13 September 1994; received in final form 30 November 1994. This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Editor Peter Dorato. Corresponding author Fahimeh Rezayat. Tel. (310) 516-3484; E-mail Rezayat@dhvx20.csudh.edu.

† School of Management, California State University, Carson, CA 90747, U.S.A.

and Cristion (1992, 1994) propose estimating the NN weights by using the output error along with the 'simultaneous perturbation' approximation of $g_k(\cdot)$ (Spall, 1992). Essentially this approximation, at each k, is based on two instantaneous observed values of loss function L_k, say $\hat{L}_k^{(\pm)}$, which are based on target vectors t_{k+1} and observed values of the state x_k to estimate the NN weights without the need to estimate or assume a separate model for the equations governing the dynamics of the system. For more on the SPSA approximation see Spall and Cristion (1994, pp. 11–13). The hth component of the gradient estimate, $\hat{g}_{kh}(\cdot)$, $h = 1, 2, \ldots, p$, is calculated via SPSA as follows:

$$\hat{g}_{kh}(\hat{\Theta}_{k-1}) = \frac{\hat{L}_k^{(+)} - \hat{L}_k^{(-)}}{2c_k\Delta_{kh}}, \quad h = 1, 2, \ldots, p, \tag{2}$$

where p represents the number of connection weights of the NN; $\Delta_k = \{\Delta_{kh}\}$ is a vector of p symmetrically distributed (about 0) i.i.d. random variables with $E(\Delta_{kh}^{-2})$ uniformly bounded. Note that these are general conditions, but that Δ_{kh} may not be either uniformly or normally distributed (it can be symmetrically Bernoulli ± 1 distributed, which we shall use in this study); $\{c_k\}$ is a sequence of positive numbers satisfying certain regularity conditions; $\hat{L}_k^{(\pm)}$ are observed loss values based on t_{k+1}, $x_{k+1}^{(\pm)}$ and $u_k^{(\pm)}$ (note that all p elements of the gradient vector use the same two loss values); $u_k^{(\pm)}$ are control values, and they are the NN output. Their values are based on the NN p-element weight vector $\hat{\Theta}_k = \hat{\Theta}_{k-1} \pm c_k\Delta_k$, t_{k+1} and $x_{k+1}^{(\pm)}$; $x_{k+1}^{(\pm)}$ are the process output based on $u_k^{(\pm)}$; the t_{k+1} are the desired target values for $x_{k+1}^{(\pm)}$. The SP gradient approximation is typically much more efficient than the standard finite-difference-based approach in terms of the amount of data required. As shown in Spall (1992), SPSA provides the same level of accuracy in estimating the NN weights with orders of magnitude fewer system measurements.

Note that the SPSA-based NN controller is implemented as an interactive program such that the number of 'control factors' can be changed by an operator as additional information becomes available. If one can identify the factor that is the source of out-of-control process (in standard ways such as brainstorming or experiment design) and provide it to the SPSA-based NN controller, the controller can then adjust the level of that factor. When it is difficult or expensive to identify the source of the problem, most often, the SPSA-based NN controller is able to compensate for the source of the problem.

2.2. *Statistical process control charts.* The statistical process control approaches are commonly used to identify the existence of variations beyond those resulting from inherent system limitations, as well as the cause of variations. When the statistical process control approaches are used (for monitoring, signalling out-of-control process and identifying the control factors) concurrently with the NN controller (for readjusting the levels of the control factors), a higher-quality performance, in general, will be reached. In this study we use control charts, a statistical process control technique, for monitoring the process and signalling variations in observed values beyond the specification range of the target value.

Statistical control charts provide a set of criteria that enable one to judge, at any given time, whether the process generates observations within an acceptable variation range. The Shewhart control charts (X-bar and R Charts) are commonly used for monitoring a production process and also for reduction of variations even within specification limits. When dealing with autocorrelated data, the traditional Shewhart control charts are not appropriate tools for monitoring the process (Alwan and Roberts, 1988). Therefore this study uses the special-cause control (SCC) chart under the assumption that the noise of the operations process are additive, and identically and independently distributed. With the SCC chart, the residuals or one-step-ahead forecast errors are plotted. This chart is based on the assumption that when the process is 'stable' and appropriate control action is applied, the residuals are random, and are i.i.d. with small variance. A stable process is a process that exhibits only variation resulting from inherent system limitations. But, if there are any disturbances to the process, the residuals will begin to show a departure from statistical control (for more on this see Wardell *et al.* (1994) and related discussions).

3. *Simulation experiments*

In this section the batch polymerization example provided by Vander Wiel *et al.* (1992) is used to study the performance of the SPSA-based NN controller. This example is used merely to examine the performance of the SPSA-based NN controller using a published example when another controller (one step feedback controller) is available for comparison.

In the batch polymerization example, intrinsic viscosity, a key quality characteristic of the polymer, is measured at the completion of each batch. Turnaround time is such that the viscosity measurement from the most recent batch produced in a given reactor is available when the reactor is prepared for a new batch, and autocorrelation exists among viscosity measurements. The level of catalyst has the highest effect on the level of viscosity. Based on the observed intrinsic viscosity deviations from its target value and the catalyst deviations from its nominal value, Vander Wiel *et al.* provided the following empirical model for the batch k:

$$x_k = 1.5u_{k-1} + \frac{1 - 0.22B}{1 - 0.8B}w_k, \tag{3}$$

where B is the backshift operator ($Bx_k = x_{k-1}$), x_k is observed viscosity deviation from a 100-unit target viscosity, u_{k-1} is catalyst deviation from nominal (50 units), $w_k \sim$ independent $N(0, \sigma_w^2)$, with $\sigma_w = 2.798$. Vander Wiel *et al.* provided the pure-one-step adjustment rule based on the minimum mean-square errors:

$$u_{k-1} = 0.8u_{k-2} - 0.4x_{k-1} \tag{4}$$

This simulation study compares the SPSA-based NN controller with the algorithm (4) for nonstationary and nonlinear systems. It will generate data according to the empirical model (3) and will assume that the SPSA controller does not have knowledge of (3). The loss function is the same as that in the Vander Wiel *et al.* study (the mean-square output deviation from the target value). The SCC chart with control limits of $\pm 3\sigma_w$ will be used concurrently with both controllers for monitoring the viscosity variations and signalling the existence of a special cause of variations.

Before performing the comparison, let us examine the learning rate of the NN controller in a no-change mode. Fifteen independent simulation runs, each of 1000 iterations, are conducted. Each simulation run represents the performance of one reactor. For every simulated reactor and every sequence of 100 simulated batches, the study calculates the number of simulated production batches whose viscosity deviations from the target value fall outside the SCC control chart limits. Figure 1 presents the average performance of the 15 simulated reactors. As the figure indicates, the percentage of batches whose viscosity deviations fall outside the SCC control chart limits decreases over time, and the rate of reduction (the SPSA-based NN controller learning) is relatively fast. Note that when (3) is the true representation of the model, one expects that the controller in (4) will perform better than the NN controller. But, since a true system model is seldom exactly known, it is appropriate to examine their relative performance for cases in which the process dynamics change or is unknown.

Now we consider a case where the systems dynamics are nonstationary. Following Vander Wiel *et al.*, we assume that, beginning with period 84, the process mean has shifted by an amount of 10.9. The SCC chart is used to monitor the viscosity variations. For this experiment also 15 independent simulation runs, each of 1000 iterations, are conducted for the two different controllers: the NN controller and the one-step controller (based on (4), which does not include knowledge of the system change). The criterion for evaluating the performance of controllers is the speed of each controller in readjusting the process performance after the change.

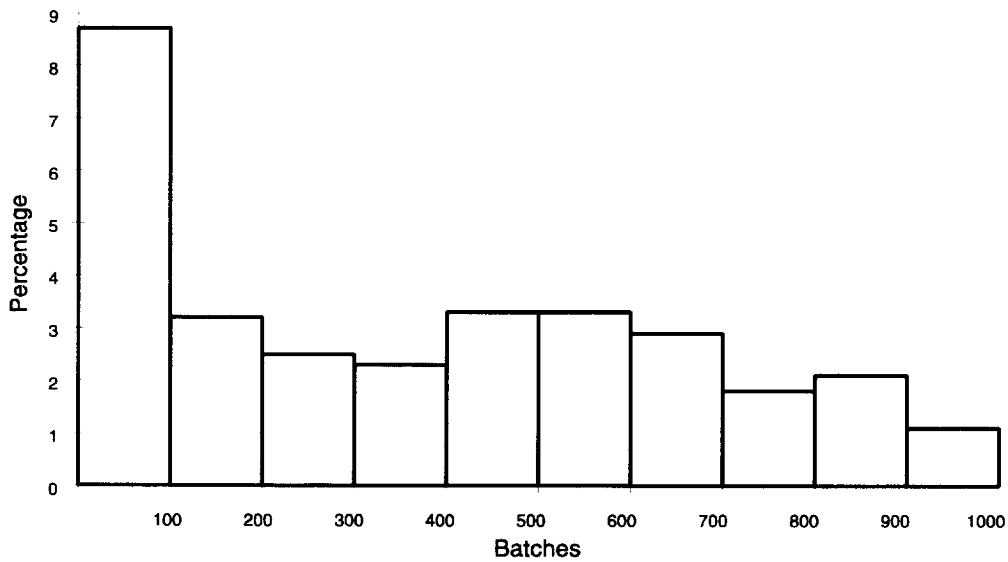

Fig. 1. Percentage of simulated viscosity deviation values that are outside the SCC chart limits.

The results of the simulation runs indicate that, for both controllers, the SCC chart provided signals almost at the same time (on average, on the 84th batch when the one-step control rule is used, and on the 84.6th batch when the SPSA-based NN controller is used). When the one-step feedback controller is used, the average RMSE value through simulated production time does not change significantly (e.g. the average RMSE for the 100th iteration is 3.11, for the 400th it is 3.14, for the 600th it is 3.23, and for the 1000th it is 3.5); whereas when the SPSA-based NN controller is used, a significant reduction of its value through the simulated production period is observed (e.g. the average RMSE for the 100th iteration is 5.58, for the 400th it is 4.75, for the 600th it is 3.55, and for the 1000th it is 2.89). These simulation results indicate when the mean has shifted, the NN controller performance is relatively close to that of the optimal one-step feedback controller, and, more importantly, it learns how to improve its performance, in contrast to the one-step controller.

To illustrate a more general case, at period 84, the system is changed to a nonlinear system (instead of just shifting the mean). In particular, it is supposed that at period 84 the structure of the process changes to

$$x_k = 0.8x_{k-1} + 0.25u_{k-1}x_{k-1} + 1.5u_{k-1} - 1.2u_{k-2} + w_k - 0.22w_{k-1}. \quad (5)$$

As above, the findings indicate that for both cases the SCC chart provided signals almost at the same time (at approximately 95th batch). The NN controller was able to compensate for the source of the problem and reduce the RMSE to 3.02 after peaking at 95th period as a result of the change in system dynamics. Not surprisingly, the controller in (4) performed poorly, because it was not able to readjust to the change in the process and caused the system to go hopelessly unstable within 10–25 iterations after the change.

On the whole, the findings of the simulation study indicate that the SPSA-based NN controller can readjust the level of inputs and perform well for distinct types of changes. Further, the SCC control chart performed well in signaling the changes and monitoring a process whose underlying structure was unknown. These offer a promising combination of techniques in quality improvement problems when system dynamics are difficult to model.

Acknowledgements—Part of this research was supported by the California State University-Dominguez Hills RSCAAP Summer Fellowship. The referees provided valuable suggestions.

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